



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\therefore \log[U + \sqrt{U^2 + \kappa}] = a + \lambda, \quad U + \sqrt{U^2 + \kappa} = e^{a+\lambda}.$$

Also, $\sqrt{U^2 + \kappa} - U = \kappa e^{-(a+\lambda)}$, $2U = e^{a+\lambda} - \kappa e^{-(a+\lambda)} = Ce^{-a} + C'e^a$, where C, C' , are constants. U not increasing indefinitely with a it follows that $C' = 0$. When a is very small, (1) becomes

$$L_{a \neq 0} U - \frac{1}{2} \pi = L_{a \neq 0} - \int_0^\infty \frac{adx}{1+x^2} = L_{a \neq 0} - \frac{a}{2} \pi = 0; \therefore C = \frac{1}{2} \pi, \text{ and}$$

$$u = - \int_0^\infty \frac{x \sin ax dx}{1+x^2} = \frac{\pi}{2} e^{-a} \quad (a \text{ being positive}).$$

$$\text{But } \int_0^\infty \frac{x \sin ax}{1+x^2} dx = \frac{\pi}{2} - u = \frac{\pi}{2} (1 - e^{-a}).$$

Differentiating with respect to a ,

$$\int_0^\infty \frac{x \cos ax}{1+x^2} dx = \frac{\pi}{2} e^{-a}.$$

\therefore etc. (Cf. Roberts' *Treatise on the Integral Calculus*, Part I, p. 181.)

317. Proposed by C. N. SCHMALL, New York City.

A generating line of a right circular cylinder passes through the center of a sphere. The diameter of the cylinder is less than the radius of the sphere. Show that the surface of the cylinder included within the sphere is given by an elliptic integral.

Solution by A. M. HARDING, Fayetteville, Arkansas.

Let a = diameter of cylinder; r = radius of sphere. Choose the generating line of cylinder for z -axis. Let equation of sphere and cylinder be

$$x^2 + y^2 + z^2 = r^2 \text{ and } x^2 + y^2 = ax,$$

respectively. Then

$$\frac{A}{4} = \int \int \left[1 + \left(\frac{\partial y}{\partial x} \right)^2 + \left(\frac{\partial y}{\partial z} \right)^2 \right]^{\frac{1}{2}} dz dx.$$

Eliminate y and obtain $z^2 + ax = r^2$. Hence z -limits are 0 and $\sqrt{r^2 - ax}$, x -limits are 0 and a .

From equation of cylinder, we find

$$\frac{\partial y}{\partial x} = \frac{a-2x}{2y}, \quad \frac{\partial y}{\partial z} = 0.$$

$$\therefore \frac{A}{4} = \int_0^a \int_0^{\sqrt{r^2 - ax}} \left[1 + \left(\frac{a-2x}{2y} \right)^2 \right]^{\frac{1}{2}} dz dx = \int_0^a \int_0^{\sqrt{r^2 - ax}} \frac{a}{2\sqrt{ax - x^2}} dz dx,$$

$$\text{since } y^2 = ax - x^2.$$

$$\therefore A = 2a \int_0^a \sqrt{\left(\frac{r^2 - ax}{ax - x^2} \right)} dx. \quad \text{Putting } x = a \sin^2 \phi,$$

$$A = 2a \int_0^{\frac{1}{2}\pi} \frac{\sqrt{r^2 - a^2 \sin^2 \phi}}{\sqrt{a^3 \sin^2 \phi - a^2 \sin^4 \phi}} \cdot 2a \sin \phi \cos \phi d\phi$$

$$= 4a \int_0^{\frac{1}{2}\pi} \sqrt{r^2 - a^2 \sin^2 \phi} \cdot \frac{a \sin \phi \cos \phi}{a \sin \phi \sqrt{1 - \sin^2 \phi}} d\phi$$

$$= 4ar \int_0^{\frac{1}{2}\pi} \sqrt{\left(1 - \frac{a^2}{r^2} \sin^2 \phi \right)} d\phi = 4ar E\left(\frac{a}{r}, \frac{\pi}{2}\right), \quad a < r.$$

Also solved by Francis Rust and J. Scheffer.

318. Proposed by JOHN C. GREGG, Greencastle, Ind.

A thread is wound spirally n times around a cone, the radius of whose base is r , and slant height h , the turns being at uniform distance apart. If the thread is kept taut, what will be the length of the trace of its end on a horizontal plane?

No correct solution of this problem has been received. Professor Feemster has given a solution finding the length of the thread. But the problem does not require that. Let us have a number of solutions of this problem. ED. F.

319. Proposed by C. N. SCHMALL, New York City.

Given $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, prove

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = 4xyz.$$

Solution by S. LEFSCHETZ, Ph. D., The University of Nebraska.

If Δ is the determinant proposed, we have: